Winter semester 2013/14 | Research Institute for Discrete Mathematics
Prof. Dr. S. Held | University of Bonn

Linear and Integer Optimization

Exercise Sheet 2

Exercise 2.1: Let P and Q be two polyhedrons in $\mathbb{K}^n$. Prove that the closure $\overline{conv(P \cup Q)}$ is a polyhedron and specify polyhedrons P, Q such that $conv(P \cup Q)$ is not a polyhedron.
Here $conv(Y) := \left\{\sum_{i=1}^{k} \lambda_i x_i : k \in \mathbb{N}, 0 \leq \lambda_i \leq 1, x_i \in Y \text{ for } 1 \leq i \leq k; \sum_{i=1}^{k} \lambda_i = 1\right\}$ is the convex hull of a set $Y \subseteq \mathbb{K}^n$. (4 Points)

Exercise 2.2: (Three equivalent characterizations of vertices)
Let $A \in \mathbb{K}^{m \times n}$, $b \in \mathbb{K}^m$, and $P = P(A, b) := \{x \in \mathbb{K}^n \ : \ Ax \leq b\}$. An element $x \in P$ is called an **extreme point** if there are no two elements $y, z \in P$ different from x ($x \notin \{y, z\}$) such that x is a convex combination of y and z, i.e. there is no $\lambda \in [0, 1]$ with $x = \lambda y + (1 - \lambda)z$. Let $x^\star \in P$. Show

1. If $x^\star$ is a vertex of P, then $x^\star$ is also an extreme point of P. (2 Punkte)
2. If there is a subsystem $A'x \leq b'$ of $Ax \leq b$ for which $A'x^\star = b'$ with $rank(A') = n$, then $x^\star$ is a vertex of P. (3 Punkte)
3. Let $x^\star$ be an extreme point of P, then there is a subsystem $A'x \leq b'$ of $Ax \leq b$ for which $A'x^\star = b'$ and $rank(A') = n$ hold. (3 Points)

Exercise 2.3: Prove that any set $X \subseteq \mathbb{K}^n$ with $|X| > n + 1$ can be decomposed into subsets X_1 and X_2 such that $\text{conv}(X_1) \cap \text{conv}(X_2) \neq \emptyset$. (4 Points)

Exercise 2.4: Let $X \subseteq \mathbb{K}^n$ and $y \in \text{conv}(X)$. Prove that one can find $x_1, \ldots, x_{n+1} \in X$ with $y \in \text{conv}(\{x_1, \ldots, x_{n+1}\})$.
(4 Points)

Submission deadline: Tuesday, 29.10.2013, before the lecture (in groups of two).