Winter term 2014/15
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Combinatorial Optimization

Exercise Sheet 12

Exercise 12.1:
Prove that the POLYMATROID GREEDY ALGORITHM, when applied to a vector $c \in \mathbb{R}^E_{>0}$ and a submodular but not necessarily monotone function $f : 2^E \to \mathbb{R}$ with $f(\varnothing) \geq 0$, solves

$$\max\left\{c^T x \mid \sum_{e \in A} x_e \leq f(A) \text{ for all } A \subseteq E\right\}.$$

4 Points

Exercise 12.2:
Prove:

(i) Let $(E, \mathcal{F})$ be a matroid, $A \in \mathcal{F}$ arbitrary, and $\mathcal{F}_A := \{X \Delta A \mid X \in \mathcal{F}\}$. Then $(E, \mathcal{F}_A)$ is a greedoid. 2 Points

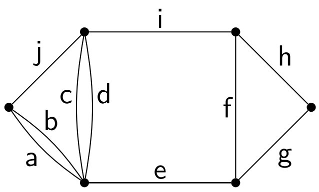
(ii) Let E be a finite set and $\mathcal{P} \subseteq 2^E$ a familiy of subsets with $\varnothing \notin \mathcal{P}$ such that $A, B \in \mathcal{P}$, $|A| = |B|$, and $|A \Delta B| = 2$ implies $A \cap B \in \mathcal{P}$. Then $(E, 2^E \setminus \mathcal{P})$ is a greedoid. 2 Points

Exercise 12.3: Let $(E, \mathcal{F})$ be a greedoid and $c' : E \to \mathbb{R}_+$. We define the bottleneck function $c(F) := \min_{e \in F} c'(e)$ for every $F \subseteq E$. Show that the GREEDY ALGORITHM FOR GREEDOIDS, when applied to $(E, \mathcal{F})$ and c, finds a base of $\mathcal{F}$ with $c(F)$ maximum. 4 Points

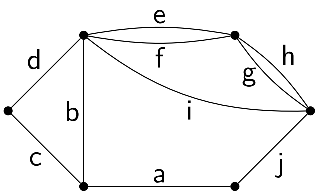
Exercise 12.4:
We call a graph *christmas tree* if it has no cycles and its edge set can be partitioned into sets $E_0 \dot{\cup} \ldots \dot{\cup} E_k$ such that $(V(E_0), E_0)$ is a path and, for $1 \leq i \leq k$, $(V(E_i), E_i)$ is a path with $|E_i| \leq 2$ and one endpoint in $V(E_0)$. A graph whose connected components are christmas trees is called festive. Prove or disprove:

(i) If $G = (V, E)$ is a graph and $\mathcal{F} := \{F \subseteq E \,|\, (V, F) \text{ is a festive graph}\}$, then $(E, \mathcal{F})$ is a matroid. 2 Point

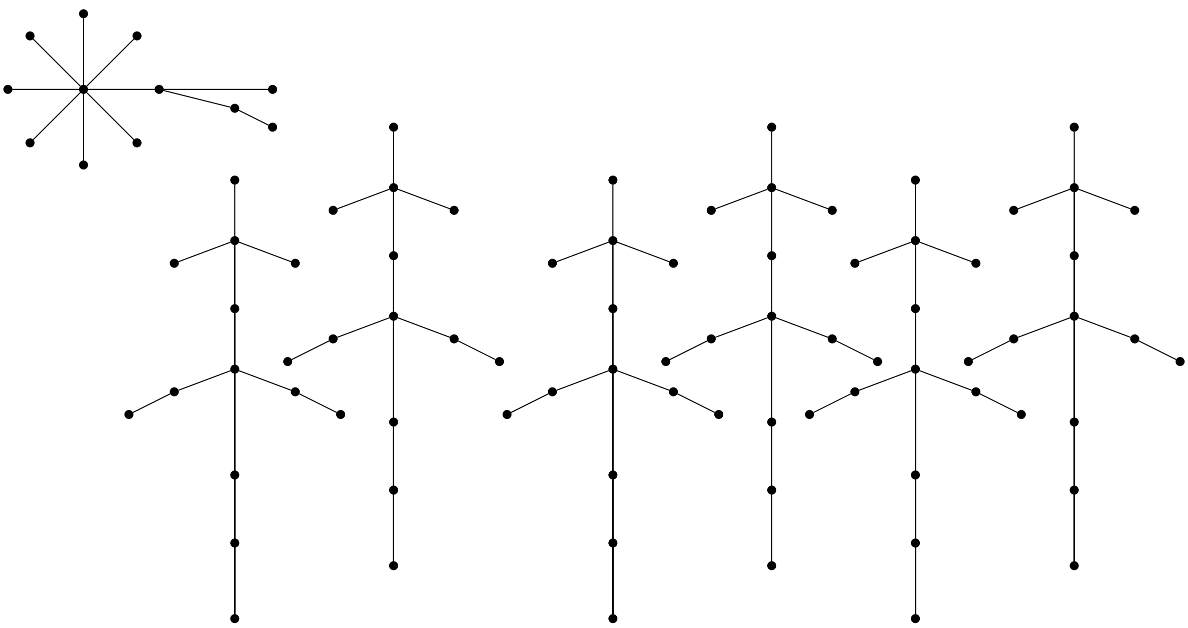
(ii) There exists a set J which is the edge set of a spanning christmas tree in both G_1 and G_2. 2 Points



G_1



G_2



A festive graph.

Deadline: Tuesday, January 13, 2015, before the lecture.
Information: Submissions by groups of one or two students are allowed.