Winter term 2014/15
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Combinatorial Optimization

Exercise Sheet 10

Exercise 10.1:
Let $G = (V, E)$ be a graph. We define $\mathcal{F} := \{X \subseteq V \mid X \text{ is covered by some matching}\}$ and $\mathcal{F}^* := \{X \subseteq V \mid X \text{ is exposed by some maximum matching}\}$.

1. Show that $(V, \mathcal{F})$ and $(V, \mathcal{F}^*)$ are matroids. (3 Points)
2. Show that $(V, \mathcal{F}^*)$ is the dual matroid of $(V, \mathcal{F})$. (2 Points)

Exercise 10.2:
Let P be the convex hull of characteristic vectors of independent sets of a matroid $(E, \mathcal{F})$. Prove that $P \cap \{x \in \mathbb{R}^E \mid \sum_{e \in E} x_e = r(E)\}$ is the convex hull of characteristic vectors of bases of E. (3 Points)

Exercise 10.3:
Let $\mathcal{M} = (E, \mathcal{F})$ be a matroid, $B \subseteq E$, and $J \subset E$ a basis of B. We define $\mathcal{M}/B := (E \setminus B, \{J' \subseteq E \setminus B \mid J' \cup J \in \mathcal{F}\})$. Prove:

1. $\mathcal{M}/B$ is a matroid that does not depend on the choice of J and the rank function of $\mathcal{M}/B$ is given by $r'(A) = r(A \cup B) - r(B)$ for all $A \subseteq E \setminus B$. (4 Points)
2. Let $\varnothing = T_0 \subseteq T_1 \subseteq \ldots \subseteq T_{l+1} = E$. The bases of T_l in $\mathcal{M}$ that intersect T_i in a basis of T_i for each $i \in \{1, \ldots, l\}$ are the bases of T_l in the matroid $\mathcal{N} := \mathcal{N}_0 \oplus \mathcal{N}_1 \oplus \ldots \oplus \mathcal{N}_l$, where $\mathcal{N}_i := (\mathcal{M}/T_i) \setminus \overline{T}_{i+1}$. (4 Points)

Deadline: Tuesday, December 16, 2014, before the lecture.
Information: Submissions by groups of one or two students are allowed.