Winter term 2014/15 Research Institute for Discrete Mathematics
Professor Dr. Stephan Held University of Bonn
Jannik Silvanus

Combinatorial Optimization

Exercise Sheet 7

Exercise 7.1: For $n \in \mathbb{N}$, let P_n be the convex hull of all even 0-1-vectors. More precisely, let

$$P_n = \operatorname{conv}\{x \in \{0,1\}^n : \sum_{i=0}^{n} x_i \equiv 0 \pmod 2\}.$$

Prove $rc(P_n) = 2^{\Theta(n)}$, i.e., P_n has an exponential relaxation complexity.

(4 Points)

Exercise 7.2: Let G be a connected graph, $T \subseteq V(G)$ with $|T|$ even, and $F \subseteq E(G)$. A subset $C \subseteq E(G)$ is called a T-*cut* if $C = \delta(U)$ for some $U \subseteq V(G)$ with $|U \cap T|$ odd. Prove:

(i) F has nonempty intersection with every T-join if and only if F contains a T-cut.

(ii) F has nonempty intersection with every T-cut if and only if F contains a T-join.

(4 Points)

Exercise 7.3: Let G be a graph with edge weights $c : E(G) \to \mathbb{R}_{>0}$. A set $F \subseteq E(G)$ is called *odd cover* if the graph which results from G by successively contracting each $e \in F$ is Eulerian. Show that it is possible in polynomial time to find an odd cover F that minimizes $c(F)$ or to decide that none exists. We use the notation $c(F) := \sum_{e \in F} c(e)$ for edge sets $F \subset E(G)$.

(4 Points)

Exercise 7.4: Show that the following algorithm finds in a graph G (which is not a forest) with edge weights $w : E(G) \to \mathbb{R}$ a cycle $C \subset E(G)$ that minimizes $\frac{w(C)}{|C|}$ in strongly polynomial time: First reduce all edge lengths by $\max\{w(e) | e \in E(G)\}$. Then find a minimum-weight $\emptyset$-join J. If $w(J) = 0$ output a cycle of length 0, otherwise add $\frac{-w(J)}{|J|}$ to all edge lengths and iterate (i.e. find again a minimum-weight $\emptyset$-join).

(4 Points)

Deadline: Tuesday, November 25, 2014, before the lecture.
Information: Submissions by groups of one or two students are allowed.