Winter term 2014/15 Research Institute for Discrete Mathematics
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Combinatorial Optimization

Exercise Sheet 6

Exercise 6.1: Let $G = (V, E)$ be an undirected graph and Q its fractional perfect matching polytope, which is defined by

$$Q = \{x \in \mathbb{R}^E : x_e \geq 0 \ (e \in E), \sum_{e \in \delta(v)} x_e = 1 \ (v \in V)\}.$$

If G is bipartite, Q is identical to the perfect matching polytope of G. Now, consider the first Gomory-Chvátal-truncation Q' of Q. Prove that Q' is always identical to the perfect matching polytope of G. (4 Points)

Exercise 6.2: Let $G = (V, E)$ be an undirected graph and $n := |V|$. Prove that the spanning tree polytope of G is in general a proper subset of the polytope

$$\{x \in [0, 1]^E : \sum_{e \in E} x_e = n - 1, \sum_{e \in \delta(X)} x_e \geq 1 \text{ for } \varnothing \neq X \subset V\}.$$

(4 Points)

Exercise 6.3: Let $G = (V, E)$ be an undirected graph and $n := |V|$. Prove that the following inequality system with $\mathcal{O}(n^3)$ variables and constraints describes a polytope whose orthogonal projection to the x-variables yields the spanning tree polytope of G.

$$\begin{aligned}
x_e &\geq 0 && (e \in E) \\
z_{u,v,w} &\geq 0 && (\{u, v\} \in E, w \in V \setminus \{u, v\}) \\
\sum_{e \in E} x_e &= n - 1 && \\
x_e &= z_{u,v,w} + z_{v,u,w} && (e = \{u, v\} \in E, w \in V \setminus e) \\
x_e + \sum_{\{u,v\} \in \delta(v) \setminus \{e\}} z_{u,v,w} &= 1 && (v \in V, e = \{v, w\} \in \delta(v))
\end{aligned}$$

(4 Points)

Note the second page!

Exercise 6.4: Let $G = (V, E)$ be an undirected graph and $n := |V|$. Prove that the convex hull of the incidence vectors of all forests in G is the polytope

$$\{x \in [0,1]^E : \sum_{e \in E(G[X])} x_e \leq |X| - 1 \text{ for } \varnothing \neq X \subseteq V\}.$$

(4 Points)

Deadline: Tuesday, November 18, 2014, before the lecture.
Information: Submissions by groups of one or two students are allowed.