Winter term 2012/13
Juniorprofessor Dr. Stephan Held
Jan Schneider

Research Institute for Discrete Mathematics
University of Bonn

Combinatorial Optimization

Exercise Sheet 13

Exercise 13.1:
Let G be an undirected graph and M a maximum matching in G.

1. Let F_1 and F_2 be two special blossom forests with respect to M, both with the maximum possible number of edges. Show that the sets of inner vertices in F_1 and F_2 are the same. (3 Points)

2. Let $\mathcal{F}$ be the familiy of those subsets $X \subseteq E(G)$ for which there exists a special blossom forest F with respect to M with $E(F) \setminus M = X$. Prove that $(E(G) \setminus M, \mathcal{F})$ is a greedoid.
Hint: Use 1. (3 Points)

Exercise 13.2:
Prove:

1. Let $(E, \mathcal{F})$ be a matroid, $A \in \mathcal{F}$ arbitrary, and $\mathcal{F}_A := \{X \Delta A \mid X \in \mathcal{F}\}$. Then $(E, \mathcal{F}_A)$ is a greedoid. (3 Points)

2. Let E be a finite set and $\mathcal{P} \subseteq 2^E$ a familiy of subsets with $\varnothing \notin \mathcal{P}$ such that $A, B \in \mathcal{P}$, $|A| = |B|$, and $|A \Delta B| = 2$ implies $A \cap B \in \mathcal{P}$. Then $(E, 2^E \setminus \mathcal{P})$ is a greedoid. (3 Points)

Exercise 13.3:
Let $(E, \mathcal{F})$ be a greedoid and $c' : E \to \mathbb{R}_+$. We define the bottleneck function $c(F) := \min_{e \in F} c'(e)$ for every $F \subseteq E$. Show that the Greedy Algorithm for Greedoids, when applied to $(E, \mathcal{F})$ and c, finds an $F \in \mathcal{F}$ with $c(F)$ maximum. (4 Points)

Deadline: Tuesday, January 22, 2013, before the lecture.