Winter term 2012/13
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Combinatorial Optimization

Exercise Sheet 11

Exercise 11.1:
Let P be a nonempty polymatroid. Show that there is a monotone function f with $f(\emptyset) = 0$ and $P = P(f)$.

(2 Points)

Exercise 11.2:
Let $\mathcal{M}_1, \mathcal{M}_2$ be matroids on E. Let B be a maximal partitionable subset with respect to $\mathcal{M}_1$ and $\mathcal{M}_2^*$, and let J_1, J_2 be an associated partitioning. Furthermore, let B_2 be a basis of $\mathcal{M}_2^*$ with $J_2 \subseteq B_2$. Show that $B \setminus B_2$ is a common independent set of $\mathcal{M}_1$ and $\mathcal{M}_2$ of maximum cardinality.

(4 Points)

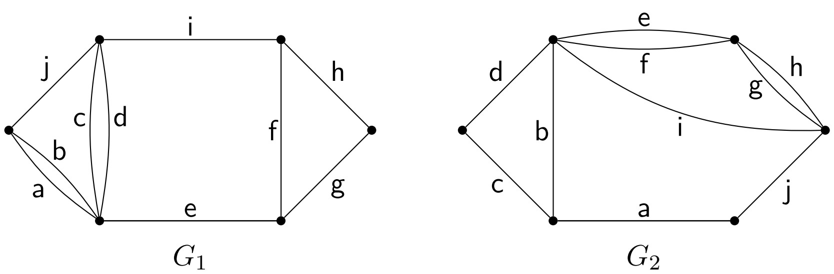
Exercise 11.3:
Let G be a connected graph and k an integer. Prove:

1. G has two edge-disjoint spanning trees if and only if there does not exist a partition of V into sets $V_0, \dots, V_p$ such that $|E(V_0, \dots, V_p)| < 2p$, where $E(V_0, \dots, V_p)$ denotes the set of edges with endpoints in different V_i.
   Hint: Apply the matroid intersection theorem to $\mathcal{M}$ and $\mathcal{M}^*$, where $\mathcal{M}$ is the cycle matroid on G. (3 Points)
2. G has k edge-disjoint spanning trees if and only if there does not exist a partition of V into sets $V_0, \dots, V_p$ such that $|E(V_0, \dots, V_p)| < kp$. (3 Points)

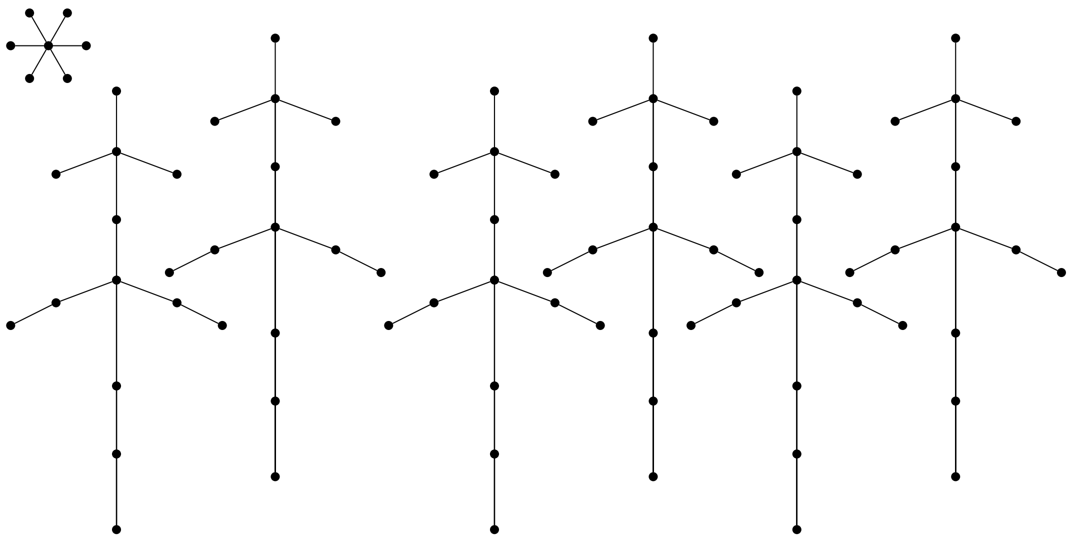
Continued on page 2.

Exercise 11.4:
We call a graph *christmas tree* if it has no cycles and its edge set can be partitioned into sets $E_0 \dot\cup \ldots \dot\cup E_k$ such that $(V(E_0), E_0)$ is a path and, for $1 \le i \le k$, $(V(E_i), E_i)$ is a path with $|E_i| \le 2$ and one endpoint in $V(E_0)$. A graph whose connected components are christmas trees is called festive. Prove or disprove:

1. If $G = (V, E)$ is a graph and $\mathcal{F} := \{F \subseteq E \,|\, (V, F) \text{ is a festive graph}\}$, then $(E, \mathcal{F})$ is a matroid. (1 Point)
2. There exists a set J which is the edge set of a spanning christmas tree in both G_1 and G_2. (4 Points)



Deadline: Tuesday, December 25, 2012, before the lecture.



A festive graph.