Winter term 2012/13
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Combinatorial Optimization

Exercise Sheet 7

Exercise 7.1:
Let G be a graph with edge weights $c : E(G) \to \mathbb{R}_{>0}$. A set $F \subseteq E(G)$ is called *odd cover* if the graph which results from G by successively contracting each $e \in F$ is Eulerian. Show that it is possible in polynomial time to find an odd cover F that minimizes $c(F)$ or to decide that none exists. We use the notation $c(F) := \sum_{e \in F} c(e)$ for edge sets $F \subset E(G)$.

(4 Points)

Exercise 7.2:
Let G be a graph and $T \subseteq V(G)$. Denote by $\nu(G,T)$ the maximum cardinality of a family of pairwise disjoint T-cuts and by $\tau(G,T)$ the minimum cardinality of a T-join.

1. Let J be a T-join. Prove: $|J| = \tau(G,T)$ if and only if $|C \cap J| \leq |C \setminus J|$ holds for every cycle C. (3 Points)
2. Let J be a T-join of minimum cardinality. Show that $\nu(G,T) = \tau(G,T)$ if and only if there exists a family of $|J|$ pairwise disjoint J-unique cuts in G. An edge set $E' \subseteq E(G)$ is called J-unique if $|E' \cap J| = 1$. (2 Points)

Consider the Edge-Disjoint Paths Problem: Given two graphs $G = (V,E)$ and $H = (V,F)$, decide if there exists a family $(P_f)_{f \in F}$ of edge-disjoint paths, where $P_{\{s,t\}}$ is an s-t-path in G. This problem is NP-complete even if $(V, E \dot\cup F)$ is planar.

3. Use this fact to show that it is NP-complete to decide if $\nu(G,T) = \tau(G,T)$ for some planar graph G and $X \subseteq V(G)$. (3 Points)

Exercise 7.3:
Show that the following algorithm finds in a graph G (which is not a forest) with edge weights $w : E(G) \to \mathbb{R}$ a cycle $C \subset E(G)$ that minimizes $\frac{w(C)}{|C|}$ in strongly polynomial time: First reduce all edge lengths by $\max\{w(e) | e \in E(G)\}$. Then find a minimum-weight $\emptyset$-join J. If $w(J) = 0$ output a cycle of length 0, otherwise add $\frac{-w(J)}{|J|}$ to all edge lengths and iterate (i.e. find again a minimum-weight $\emptyset$-join).

(4 Points)

Deadline: Tuesday, November 27, 2012, before the lecture.