

## Combinatorial Optimization

### Exercise Sheet 5

#### Exercise 5.1:

Let  $G = (V, E)$  be a graph. A set  $\mathcal{H} = \{S_1, \dots, S_k, v_1, \dots, v_r\}$  has property A if

- $S_i \subseteq V$  and  $|S_i|$  is odd for  $1 \leq i \leq k$ ,
- $v_i \in V$  for  $1 \leq i \leq r$ , and
- for each  $e \in E$  either  $e \subseteq S_i$  for some  $i \in \{1, \dots, k\}$  or  $v_i \in e$  for some  $i \in \{1, \dots, r\}$ .

The weight of a set  $\mathcal{H}$  with property A is  $w(\mathcal{H}) := r + \sum_{i=1}^k \frac{|S_i|-1}{2}$ . Prove

$$\nu(G) = \min\{w(\mathcal{H}) \mid \mathcal{H} \text{ is a set with property A}\}.$$

(4 Points)

#### Exercise 5.2:

Suppose that two workers have to carry out a number of jobs. Both workers need 1 hour for each job, and there are certain jobs that need to be done before certain other jobs. The task is to get all jobs done as early as possible.

This can be modeled as an acyclic directed graph  $G = (V, E)$  where an edge  $e = (i, j)$  means that job  $i$  has to be finished before job  $j$  is started. Let  $E' := \{\{i, j\} \subseteq V \mid \text{There is neither an } i\text{-}j\text{-path nor a } j\text{-}i\text{-path in } G\}$  and set  $H := (V, E')$ . Prove:

1. The workers can finish their work after  $|V| - \nu(H)$  hours. (1 Points)
2. The workers cannot be faster. (3 Points)

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**Exercise 5.3:**

Let  $G$  be an undirected graph with edge weights  $c : E(G) \rightarrow \mathbb{R}_{>0}$ , and let  $v, w \in V$  be two distinct vertices. Describe a polynomial-time algorithm that computes, among all  $v$ - $w$ -paths having an even number of edges, a path of minimum weight, and prove its correctness.

Hint: Use that minimum-weight perfect matchings can be computed in polynomial time.

(4 Points)

**Exercise 5.4:**

Show how the following problem can be solved in polynomial time: Given a graph  $G$  and edge weights  $c : E(G) \rightarrow \mathbb{R}_{>0}$ , find an edge cover  $F \subseteq E(G)$  that minimizes  $\sum_{e \in F} c(e)$ .

(4 Points)

**Deadline:** Tuesday, November 13, 2012, before the lecture.