Winter term 2012/13
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Combinatorial Optimization

Exercise Sheet 2

Exercise 2.1:
Prove: An undirected graph G is 2-edge-connected if and only if $|E(G)| \geq 2$ and G has an ear-decomposition. (2 Points)

Exercise 2.2:
Prove: Every 3-regular simple graph with at most two bridges has a perfect matching. (3 Points)

Exercise 2.3:
Let $G = (V, E)$ be a bipartite graph with bipartition $V = \{a_1, \dots, a_k\} \dot{\cup} \{b_1, \dots, b_k\}$. For any vector $x = (x_e)_{e \in E}$ we define a matrix $M_G(x) = (m^x_{ij})_{1 \leq i,j \leq k}$ by

$$m^x_{ij} = \begin{cases} x_e & \text{if } e = \{a_i, b_j\} \in E \\ 0 & \text{otherwise.} \end{cases}$$

Its determinant $\det M_G(x)$ is a polynomial in x. We further define the permanent of a $k \times k$ matrix M as

$$\operatorname{per}(M) := \sum_{\pi \in S_k} \prod_{i=1}^{k} m_{i\pi(i)},$$

where S_k is the set of permutations of $\{1, \dots, k\}$. Prove:

1. G has a perfect matching if and only if $\det M_G(x)$ is not identically 0. (3 Points)
2. If G is simple, it has exactly $\operatorname{per}(M_G(1, \dots, 1))$ perfect matchings. (4 Points)

Exercise 2.4:
Let $G = (V, E)$ be a bipartite graph with bipartition $V = A \dot{\cup} B$. Prove that there exists a forest $F = (V, E')$ with $E' \subseteq E$ and $|\delta_F(v)| = 2$ for all $v \in A$ if and only if $|\Gamma_G(X)| \geq |X| + 1$ holds for every non-empty $X \subseteq A$. (4 Points)

Deadline: Tuesday, October 23, 2012, before the lecture.