Winter term 2012/13
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Combinatorial Optimization

Exercise Sheet 1

Exercise 1.1:
Prove that for each Graph G the inequality $\tau(G) \leq 2\nu(G)$ holds and that the inequality is sharp. (2 Points)

Exercise 1.2:
Let M_1 and M_2 be two maximal matchings in a graph G. Prove that $|M_1| \leq 2|M_2|$. (2 Points)

Exercise 1.3:
Let G be a bipartite graph such that for each proper subset $F \subsetneq E(G)$ and $G' := (V(G), F)$ we have $\tau(G') < \tau(G)$. Prove that $E(G)$ is a matching. (4 Points)

Exercise 1.4:
Let G be a graph. Prove following equalities:

1. $\alpha(G) + \tau(G) = |V(G)|$ for any graph G. (1 Points)
2. $\nu(G) + \zeta(G) = |V(G)|$ for any graph G with no isolated vertices. (2 Points)
3. $\zeta(G) = \alpha(G)$ for any bipartite graph G with no isolated vertices. (1 Points)

Deadline: Tuesday, October 16, 2012, before the lecture.
Information:

- submissions by groups of one or two students are allowed.
- first exercise class: Thursday, October 19, 2012
  (presumably before or after the lecture).